COEFFICIENT OF VARIATION OF IN SITU TESTS IN SAND

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Abstract

For a research project on the behavior of piles in sand for the
Federal Highway Administration and the United States Geological
Survey, ten sites were selected where detailed load tests had been
performed on instrumented piles. The soil data at each site was
collected and consisted mainly of in situ tests including Standard
Penetration Tests, Cone Penetrometer Tests, Pressuremeter Tests, Cross
Hole Shear Wave Velocity Tests.

The standard deviation, mean and coefficient of variation were
calculated for each in situ test results. The results show that the
coefficient of variation in the vertical direction is equal to approx-
imately 1.5 times the one in the horizontal direction, that the coef-
ficient of variation for the SPT, CPT and PMT results are similar,
while it is much lower for the cross hole test.

Introduction

For a research project on the behavior of piles in sand for the
Federal Highway Administration and the United States Geological Sur-
vey, ten sites were selected where detailed load tests had been
performed on instrumented piles (1). The soil data at each site was
collected and consisted mainly of in situ tests including Standard
Penetration Tests (SPT), Cone Penetrometer Tests (CPT), Pressuremeter
tests (PMT), Cross Hole Shear Wave Velocity Tests (SWVT) (12).

Because the variability of soil properties across a site is a major
factor influencing the accuracy of pile capacity predictions, a
statistical analysis of the available soil data was performed. The
two terms used to characterize the data are the mean, μ, and the
standard deviation, σ. They are defined as

$$\mu = \frac{\sum\limits_{i=1}^{n} (X_i)}{n} \quad \dots \dots \dots \dots \dots \dots \dots \dots \quad (1)$$

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$$\text{and} \qquad \sigma = \frac{\sum\limits_{i=1}^{n} (X_i - \mu)^2}{n - 1} \qquad \cdots \cdots \cdots \cdots \cdots \cdots \quad (2)$$

where X is the data to be analyzed and n is the total number of data points entered. The ratio of the standard deviation over the mean is the coefficient of variation. This coefficient gives an indication of the scatter in the data.

Data Base

The pile load tests and the corresponding soil tests available at each site are shown on Table 1. As can be seen from that table the most common test at those sites was the SPT, then came the CPT followed by the PMT and the SWVT. The sands varied from very loose to very dense and from very fine to very coarse.

Standard Penetration Test (SPT)

The standard penetration test was performed at eight of the ten sites considered. At one of these sites, the Corpus Christi site, the Texas Highway Department dynamic penetration test was performed and the results were converted to SPT N values. Examples of SPT profiles are presented in Figs. 1 and 2.

Vertical Analysis

First, a vertical analysis was performed (12). This analysis consisted of taking the mean and standard deviation of the blow count values for each boring separately. Then all the borings at one site were analyzed together to determine the variation across the entire site. The coefficient of variation at each site ranged from 0.164 to 1.148 with an average for all the sites 0.707.

Horizontal Analysis

Due to a lack of data at some sites, a horizontal analysis could be performed at only four sites. This analysis was performed by taking all the blow count values within a certain layer across the site and computing the mean and standard deviation for that layer (12). The coefficient of variation at each site varied from 0.144 to 0.770 with an average for all sites of 0.421.

Two horizontal analyses were performed at the Lock and Dam 26 Ellis Island site. One used 20 borings along a line approximately 3000 feet (914 m) long parallel to the axis of the river. The other used 13 borings in an area 400 feet by 200 feet (122 m x 61 m). The results show (Table 2) that the coefficient of variation increases with the area considered and with the testing depth.

The coeficient of variation of the SPT data in the horizontal direction is equal to 0.59 times the coefficient of variation in the

TABLE 1.- Sites and Soil Data Available

| Site (1)	Piles (2)	SPT (3)	CPT (4)	PMT (5)	Other (6)	Reference (7)
Lock & Dam 4, Arkansas River (1963)	Steel Pipe Steel H	X				3,6
Low Sill Structure Old River, La. (1956)	Steel Pipe Steel H	X				7
Ogeechee River	Steel Pipe	X	X		Density	13,14
Lock & Dam 26, Replacement Site (1972)	Steel H	X				4
West Seattle Freeway Bridge (1980)	Octogonal Concrete	X			Self-boring PMT	9,10
Tavenas (1970)	Steel H Hexagonal Concrete	X	X			11
Gregersen (1969)	Circular Concrete	X	X			5
Corpus Christi (1971)	Square Concrete				Texas Highway Cone	2
Sellgren (1981)	Square Concrete			X		8
Lock & Dam 26 Ellis Island (1978)	Timber	X	X	X	Shear Wave Velocity	15,16
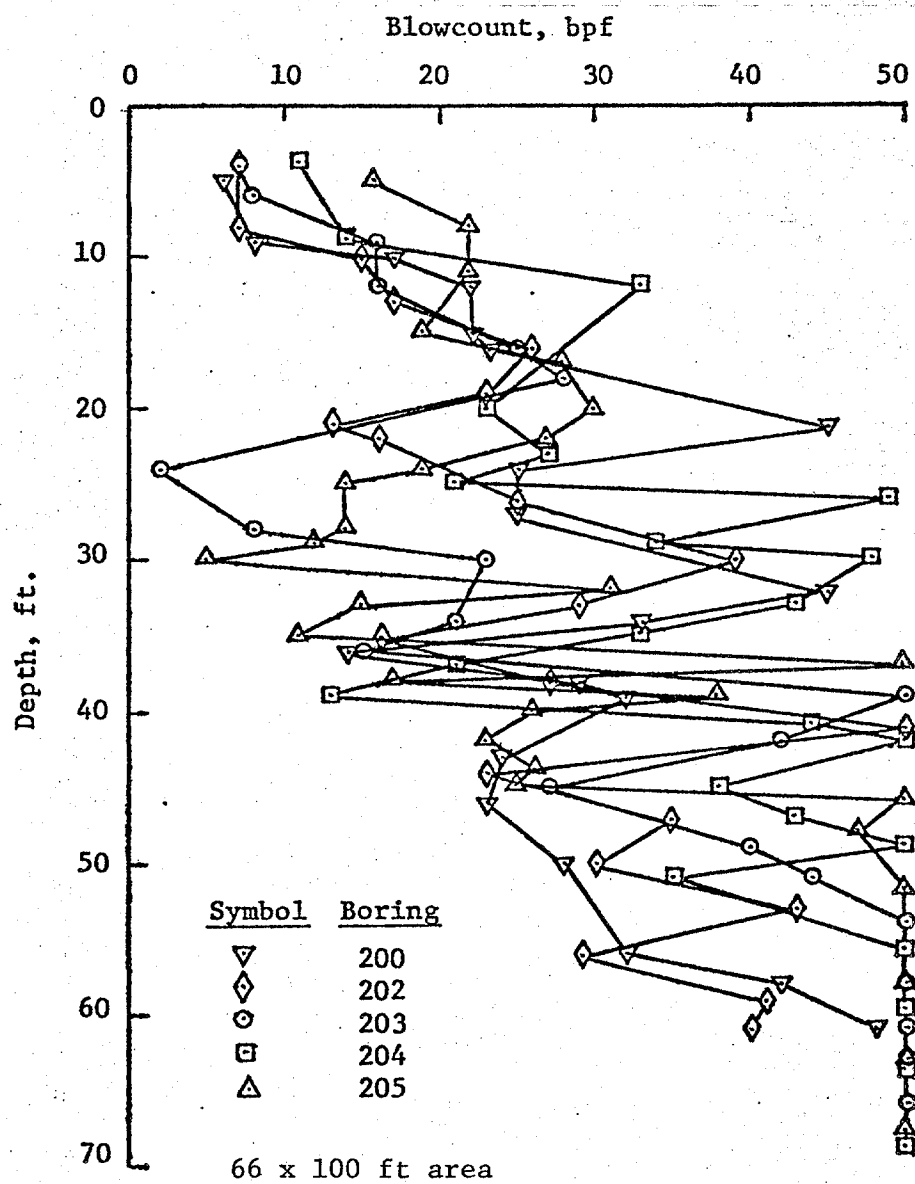
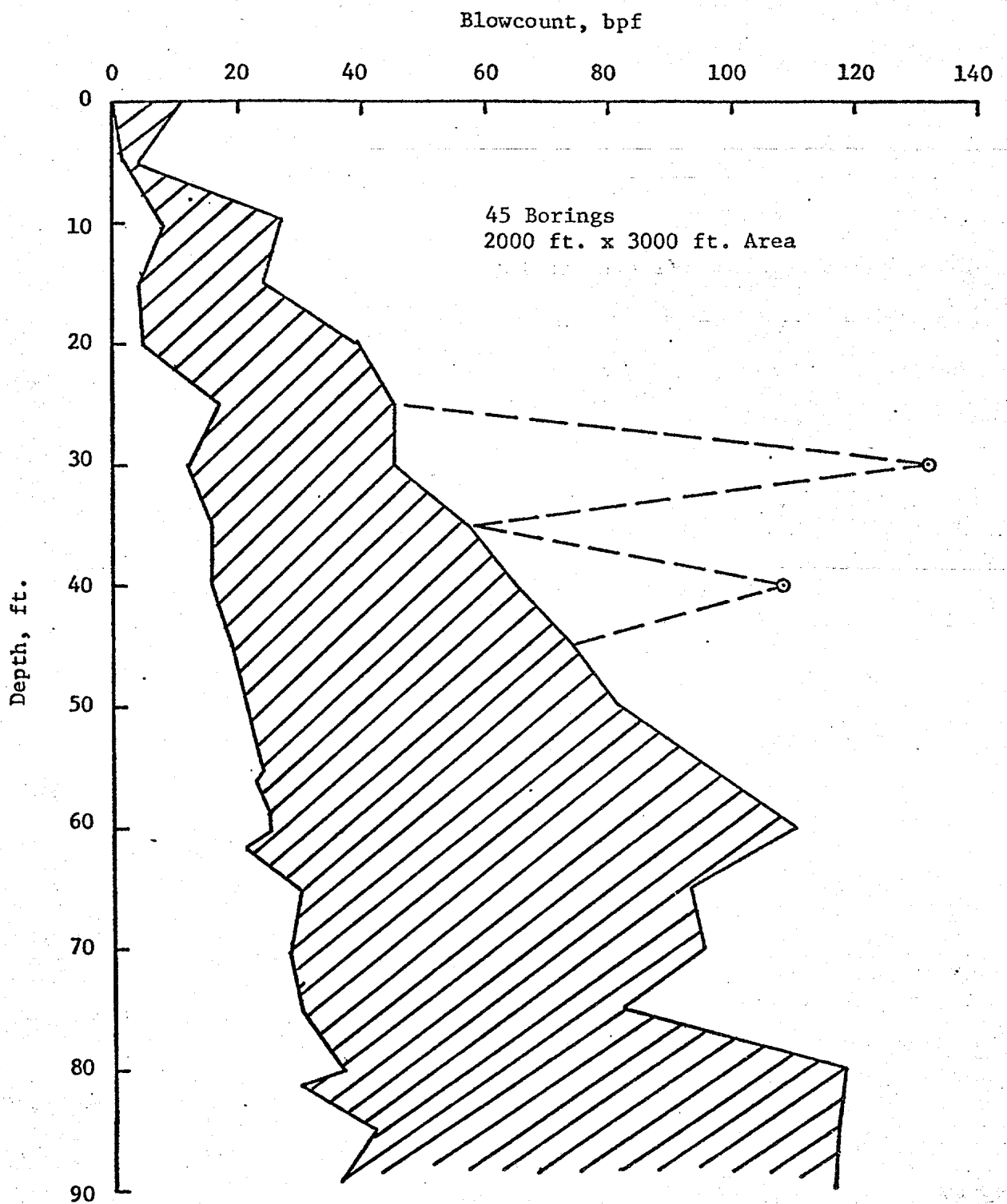FIG. 1.- Lock and Dam 4, Arkansas River: SPT
Data (1ft = 0.3048 m)

FIG. 2 .- Lock and Dam 26, New Dam Site: SPT Data
(1 ft. = 0.3048 m)

TABLE 2.- Horizontal Analysis of SPT Data

SITE	DEPTH ft.	MEAN bpf	STD.DEV. bpf	$\dfrac{\sigma}{\mu}$
Lock and Dam 26, New Dam Site, 400 ft. x 200 ft. Area	0-15	57.33	35.80	0.624
	15-20	14.70	5.23	0.356
	20-25	13.83	4.62	0.335
	25-30	17.31	3.20	0.185
	30-35	19.54	6.20	0.317
	35-40	21.15	5.97	0.274
	40-45	26.57	7.73	0.291
	45-50	28.85	6.88	0.238
	50-55	33.60	14.78	0.440
	55-60	27.33	10.20	0.373
	60-65	36.15	16.08	0.445
	65-70	44.82	20.92	0.467
	70-75	48.14	18.72	0.389
	75-80	63.17	22.42	0.355
	80-90	74.13	40.92	0.552
			Avg. =	0.376
Lock and Dam 26, New Dam Site, 3000 ft. line along river axis	0- 5	-	-	-
	5-10	43.50	39.80	0.915
	10-15	14.10	5.74	0.407
	15-20	14.00	4.15	0.296
	20-25	22.40	11.65	0.520
	25-30	27.00	25.86	0.958
	30-35	25.00	11.51	0.460
	35-40	36.57	21.33	0.583
	40-45	33.35	17.03	0.511
	45-50	40.86	17.94	0.439
	50-55	44.39	24.94	0.562
	55-60	33.72	12.52	0.371
	60-65	48.16	21.37	0.444
	65-70	52.00	21.11	0.406
	70-75	42.76	16.46	0.385
	75-80	51.50	28.05	0.545
	80-85	64.70	33.30	0.515
	85-90	56.14	34.47	0.614
			Avg. =	0.525

vertical direction. This is to be expected since sand is normally deposited in layers and generally increases in strength with depth.

Influence of Number of Borings

At the site of the new Lock and Dam 26 on the Mississippi River there were about 400 borings available, spread over a very large area. An area 400 feet by 200 feet (122 m x 61 m) was chosen, in which there were 13 SPT borings done with a 3-inch sampler. An analysis was performed to determine the influence of the number of borings on the mean and standard deviation. Borings were selected from the 13 borings in a random fashion and the mean and standard deviation were computed (12). Fig. 3 shows that the mean value becomes almost constant after six borings. Fig. 4 shows that the scatter in the data steadily decreases after two borings. This analysis is definitely not general in application but does point out that there is a certain number of borings after which it is not cost effective to perform more SPT tests. This number may, however, be site specific and may thus only be obtained through experience in that locality.

Pressuremeter Tests (PMT)

Pressuremeter test results were available at three of the ten sites. The pressuremeter data at the West Seattle Freeway site however, was insufficient for use in this study. The pressuremeter test yields two main properties of the soil: the limit pressure, p_l, and a modulus of elasticity, E_{PMT}. Example profiles are presented in Fig. 5.

Vertical Analysis

The vertical analysis was done in a similar manner to that of the SPT data. The results are shown in Table 3. The range in the coefficient of variation of p_l is from 0.261 to 0.575 with an average of 0.503 for all the borings. For E_{PMT} the coefficient of variation ranges from 0.516 to 0.783 times the mean with an average of 0.619 for all the borings. The scatter in the modulus obtained from the pressuremeter is higher than that of the limit pressure. The modulus is used in the calculation of settlement, whereas the limit pressure is used in computing the pile capacity.

Horizontal Analysis

A horizontal analysis was possible only at the Lock and Dam 26 Ellis Island site. The results are shown in Table 4. As with the SPT, the pressuremeter data shows less scatter in the horizontal direction than in the vertical. The range in the coefficient of variation for p_l is from 0.225 to 0.499 with an average of 0.399. The range for E_{PMT} is from 0.282 to 0.682 with an average of 0.416. The data is too limited to support any general conclusions. It is of interest, however, to note that the average scatter of the PMT data and that of the SPT data is approximately the same. Also, for comparison, the PMT data and the SPT data for the Lock and Dam 26
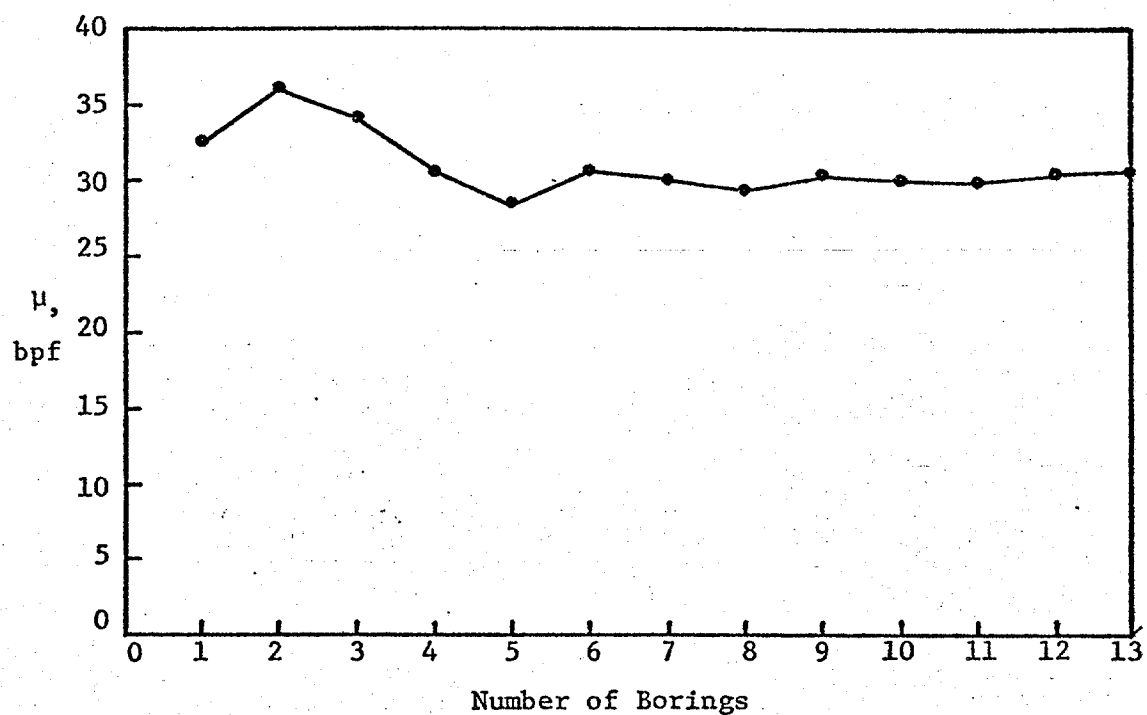
FIG. 3 - Mean of SPT Values vs. Number of Borings
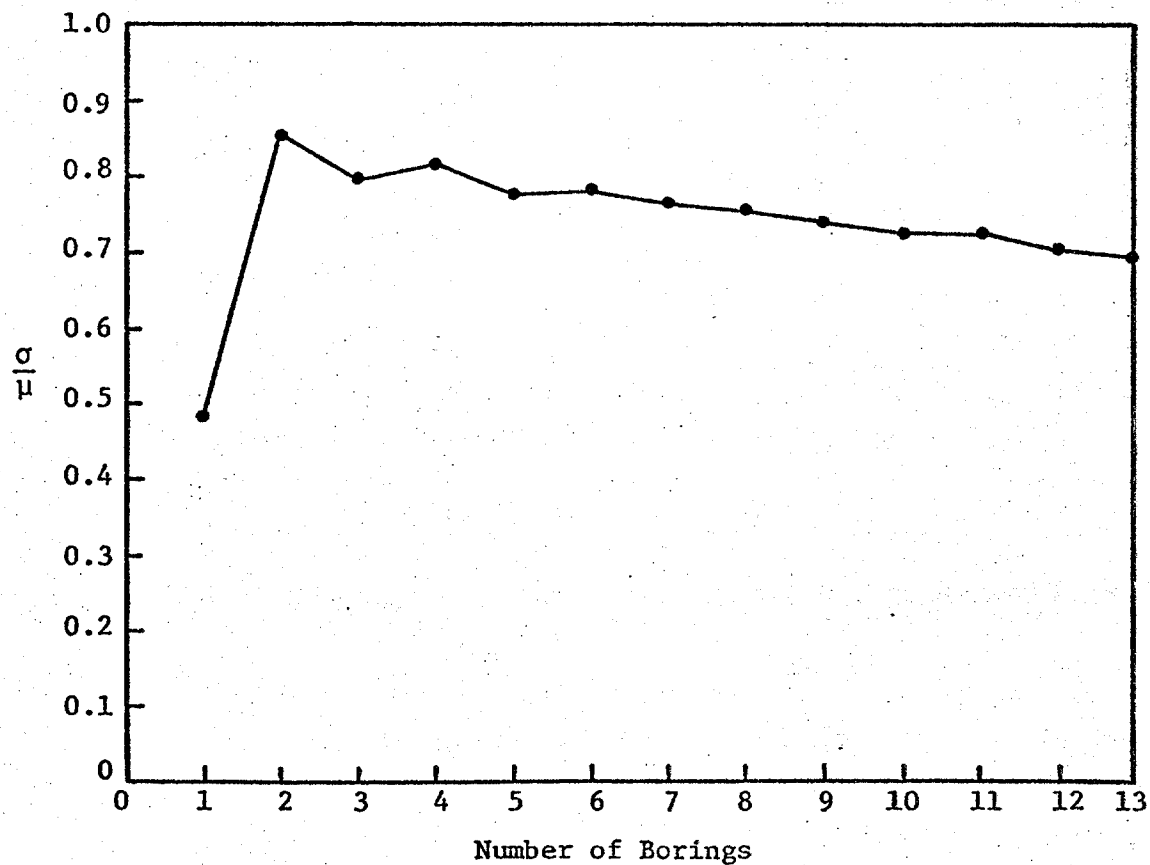(1ft. = 0.3048 m)

FIG. 4 - Ratio of Standard Deviation to Mean for
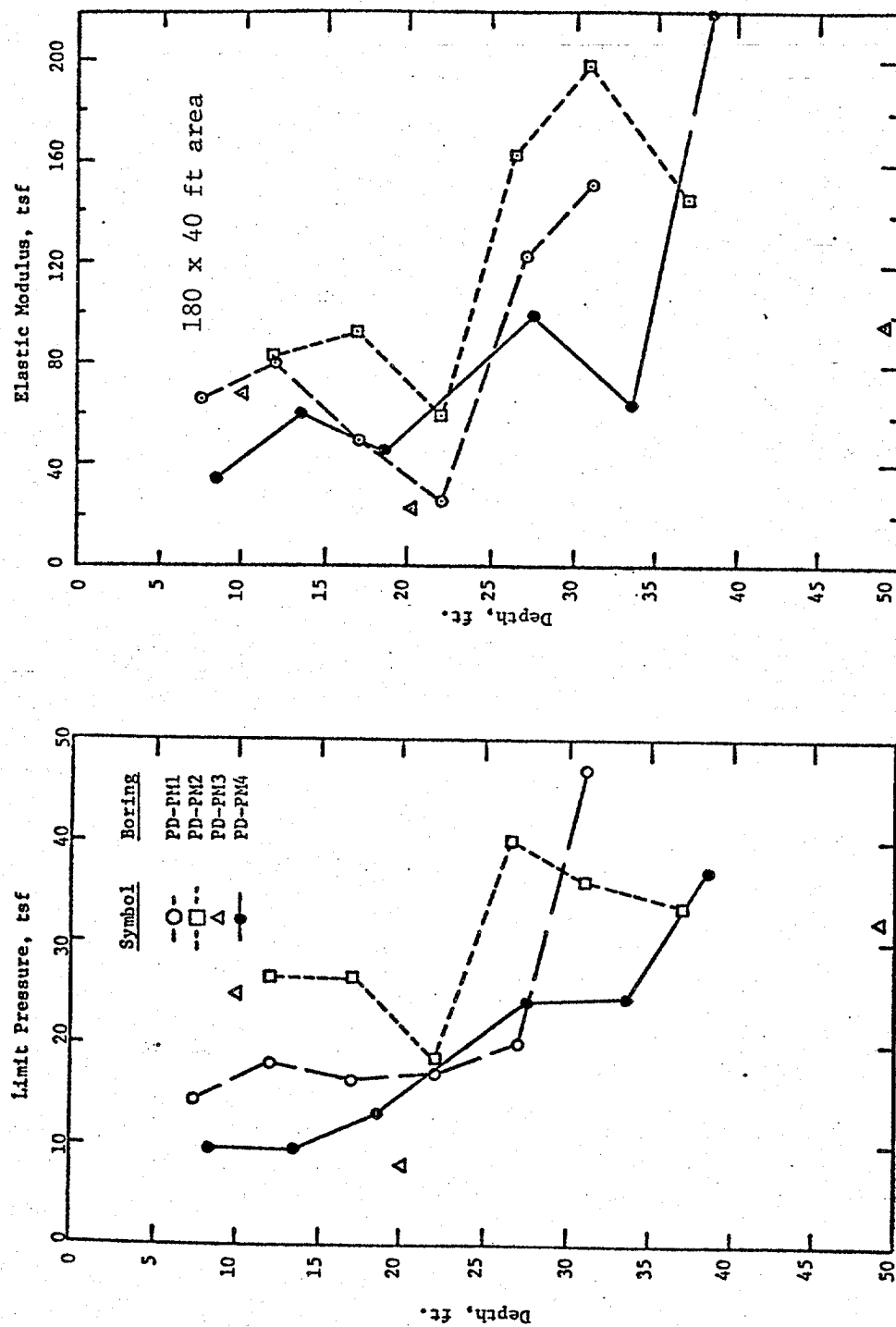SPT Values vs. Number of Borings

FIG. 5.— Lock and Dam 26 Ellis Island Site: PMT Data (from ref.(15)
(1 ft = 0.3048 m; 1 tsf = 95.76 kPa)

TABLE 3.- Vertical Analysis of PMT Data
(1 tsf = 95.76 kPa)

SITE	BORING	LIMIT PRESSURE			MODULUS		
		MEAN tsf	STD.DEV. tsf	$\frac{\sigma}{\mu}$	MEAN tsf	STD.DEV. tsf	$\frac{\sigma}{\mu}$
Lock and Dam 26, Ellis Island	PD-PM1	22.08	12.36	0.560	82.42	46.99	0.570
	PD-PM2	30.15	7.87	0.261	112.00	57.81	0.516
	PD-PM3	21.83	12.55	0.575	62.72	38.00	0.606
	PD-PM4	19.58	10.90	0.556	87.08	68.22	0.783
	All	23.64	10.86	0.460	90.34	54.88	0.607
Sellgren		89.60	50.40	0.562	14.20	8.78	0.619

TABLE 4.- Horizontal Analysis of PMT Data
(1 tsf = 95.76 kPa)

SITE	DEPTH ft	LIMIT PRESSURE			MODULUS		
		MEAN tsf	STD.DEV. tsf	$\frac{\sigma}{\mu}$	MEAN tsf	STD.DEV. tsf	$\frac{\sigma}{\mu}$
Lock and Dam 26,	0-10	16.17	7.97	0.493	52.43	16.15	0.315
Ellis Island	10-20	18.27	6.88	0.377	68.00	19.20	0.282
Site	20-30	21.25	10.60	0.499	82.53	52.29	0.682
	30-40	35.66	8.04	0.225	155.66	59.73	0.384
				Avg. = 0.399			Avg. = 0.416

Ellis Island site were obtained in the same borehole and the scatter in the data is comparable in both the horizontal and vertical directions.

Cross-Hole Shear Wave Velocity

Cross-hole shear wave velocity test data was also available at the Lock and Dam 26 Ellis Island site (Fig. 6). The results of the vertical analysis are presented in Table 5, the horizontal analysis in Table 6. The soil shear modulus, G, is related to the shear wave velocity, V_s, by:

$$G = \frac{\gamma_t}{g} V_s^2 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (3)$$

where γ_t is the total unit weight of the soil and g is the gravitational acceleration. Using the second order approximation of the Taylor series expansion for expected values, the coefficient of variation, $\frac{\sigma}{\mu}$, for G is:

$$\left(\frac{\sigma}{\mu}\right)G = \frac{2\left(\frac{\sigma}{\mu}\right)V_s}{1 + \left(\frac{\sigma}{\mu}\right)^2 V_s} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (4)$$

The average coefficient of variation for G is therefore 0.327 in the vertical direction and 0.259 in the horizontal direction. This represents a scatter in the data which is 40 percent less than the SPT and PMT data at this site.

Soil Density

The total and dry densities of the soil were measured at three of the ten sites. At the Arkansas River and Gregersen sites the measurements were made in the laboratory on samples. These measurements may therefore be influenced by disturbances due to sampling. The measurements at the Ogeechee River site were made in situ with a nuclear probe, and may be less affected by disturbance. An example of density profile is shown in Fig. 7 and the results of the analysis are given in Table 7. Only a vertical analysis was performed due to the small quantity of data at each site. The coefficient of variation is much smaller, averaging 0.057, than that of the other tests analyzed. However, the strength of the soil is much more sensitive to changes in the density than changes in the other parameters.

Static Cone Penetration Test (CPT)

Static cone penetration tests results were available at five sites. An example of the data is shown in Fig. 8. No analysis of this data was performed due to the continuous nature of the readings.

TABLE 7.- Analysis of Total and Dry Density
(1 lb/ft^3 = 16.02 kg/m^3)

SITE	BORING		MEAN lb/ft^3	STD.DEV lb/ft^3	$\dfrac{\sigma}{\mu}$
Lock and Dam 4, Arkansas River	201	γ_d	103.40	7.40	0.072
Gregersen		γ_t	121.60	4.51	0.037
		γ_d	90.81	14.08	0.155
Ogeechee River	N1	γ_t	125.58	4.55	0.036
		γ_d	102.50	3.86	0.038
	N2	γ_t	125.75	6.63	0.053
		γ_d	103.61	3.83	0.037
	N3	γ_t	125.61	5.77	0.046
		γ_d	103.33	3.92	0.038

Note: γ_d = dry density
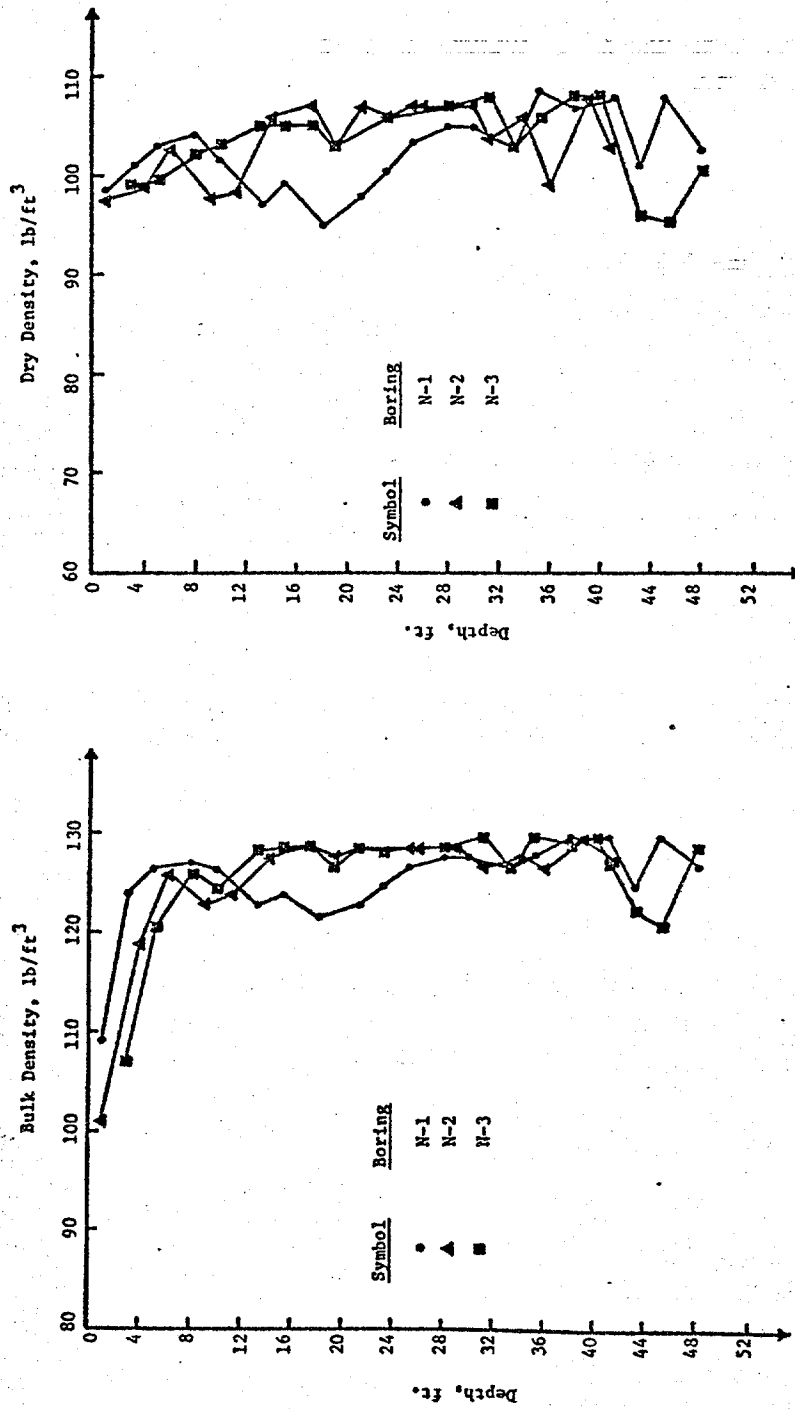γ_t = total density

80 x 20 ft area

FIG. 7. — Ogeechee River Site: Bulk and Dry Density
(1 ft. = 0.3048m; 1 lb/ft^3 = 16.02 kg/m^3)

TABLE 5.- Vertical Analysis of Cross-hole Shear Wave Data
(1 ft/sec = 30.48 cm/sec)

SITE	BORING	MEAN ft/sec	STD.DEV. ft/sec	$\frac{\sigma}{\mu}$
Lock and Dam 26, Ellis Island Site	PD-S6,S7	607.3	105.2	0.173
	PD-S9,S10	675.1	103.9	0.154
	PD-S1,S2	582.2	98.7	0.170
	PD-S4,S5	580.7	102.1	0.176
	All	607.0	105.4	0.174
			Avg. =	0.168

TABLE 6.- Horizontal Analysis of Cross-hole Shear Wave Data

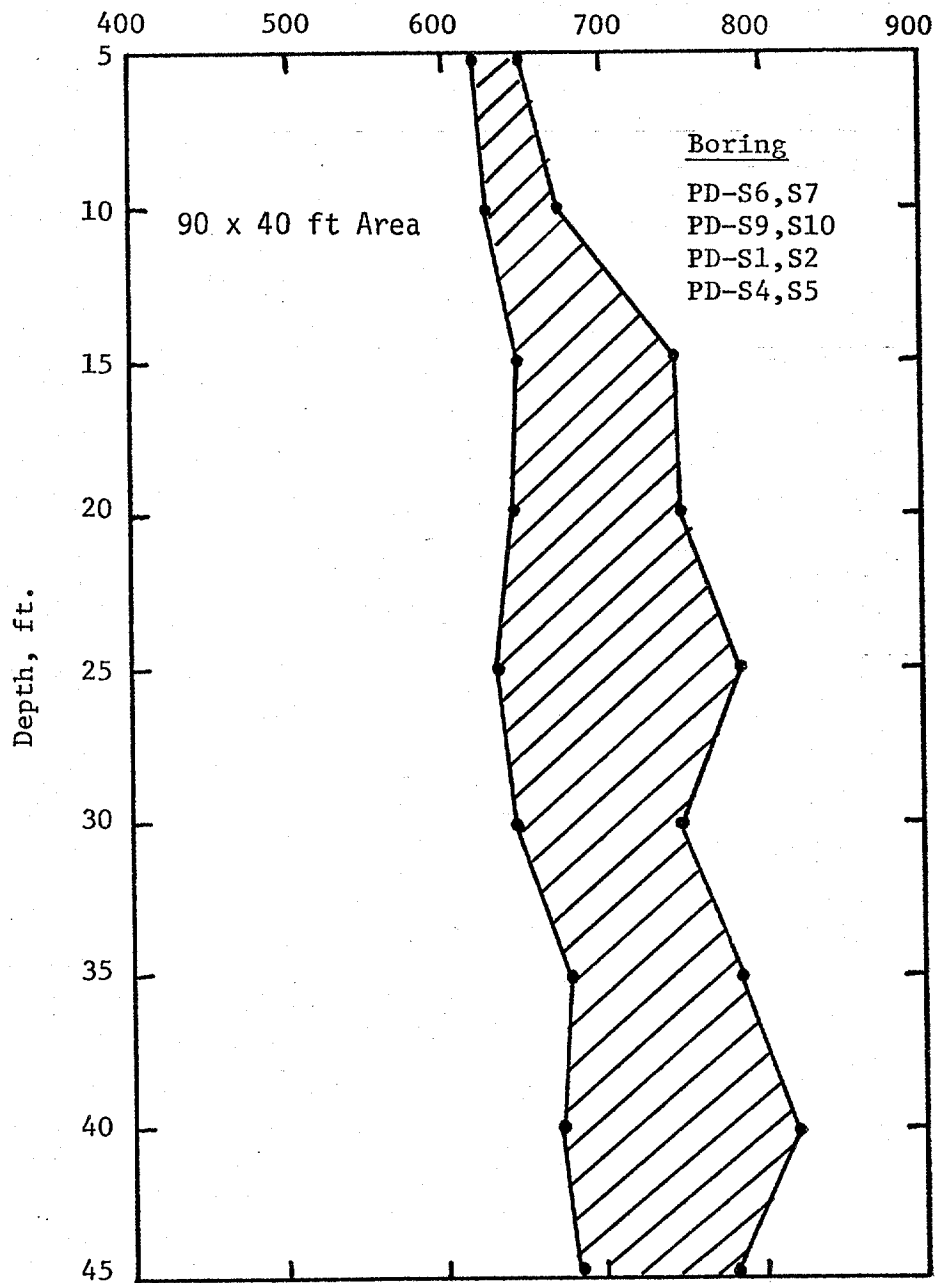| SITE	DEPTH ft	MEAN ft/sec	STD.DEV. ft/sec	$\frac{\sigma}{\mu}$
Lock and Dam 26, Ellis Island Site	5	472.0	22.1	0.047
	10	514.5	44.4	0.086
	15	606.3	92.7	0.153
	20	554.2	88.6	0.160
	25	640.6	132.5	0.207
	30	603.0	78.0	0.129
	35	684.6	82.6	0.121
	40	681.0	136.5	0.200
	45	664.5	54.5	0.082
			Avg. =	0.132

FIG 6.– Lock and Dam 26, Ellis Island Site: Crosshole
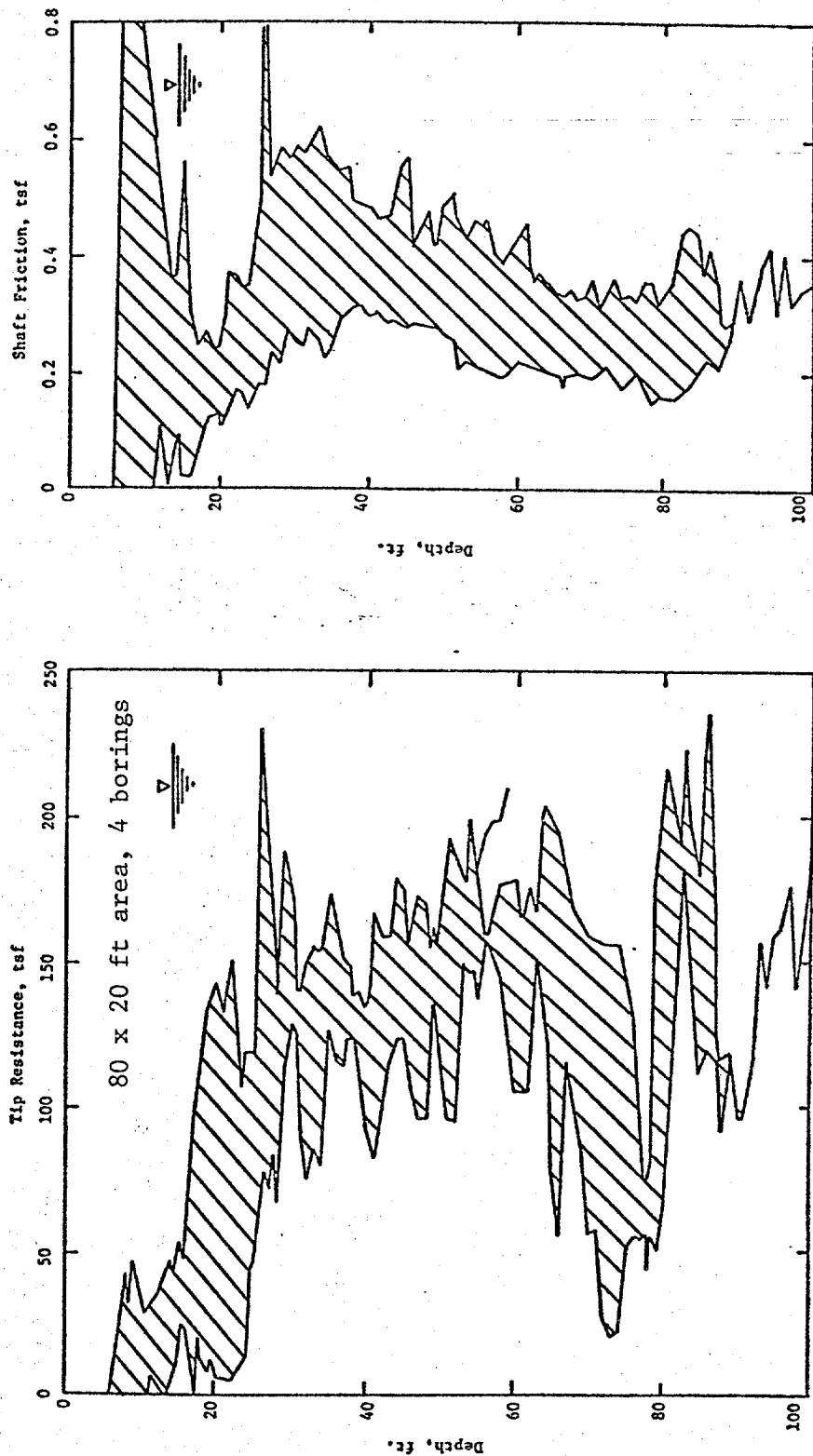Shear Wave Data (from ref. 15) (1 ft. = 0.3048;
1 ft/sec = 30.48 cm/sec.)

FIG. 8.— Ogeechee River Site: CPT Data (from ref. 13) (1 tsf = 95.76; 1 ft. = 0.3048 m)

However, a comparison of the range of CPT results to that of the SPT and PMT results shows generally the same scatter for all three type of tests.

Discussion

The coefficients of variation for the various soil tests discussed are summarized in Table 8. This table is somewhat misleading. The coefficient of variation is low for the density measurements but soil parameters are very sensitive to changes in density. The pressure-meter test and the cone penetrometer tests do not show a coefficient of variation significantly lower than the Standard Penetration Test. However, the repeatability of the SPT test from one crew to another and from one type of hammer to another has not been considered in this statistical analysis. Indeed the coefficients of variation were calculated at each site where the same crew and hammer were involved. There is little doubt that if repeatability was included the cone penetrometer would come first. The small strain shear moduli obtained from shear wave velocity tests show a smaller scatter than other para-meters.

Another consideration is to evaluate the degree of dependency between the ultimate capacity of the pile and the soil parameter measured. In this respect the fact that the SWVT provides less scatter is of little help to the engineer. Yet another point to consider is the unit cost of each test.

Precision of Pile Capacity Prediction Methods

In the process of predicting the ultimate capacity of a pile, a number of errors occur. These are: 1. The error due to the natural variability of the soil; this is tied to the fact that the pile load test and the soil test are not performed at the same location. 2. The error in testing of the soil; this is for example the error on the N value associated with the Standard Penetration Test. 3. The error in the design method; this relates for example to using the blow count N for design purposes when N may not be entirely related to the ultimate capacity of a pile; this error could also be due to simplifying assumptions for a theoretical method. 4. The error in the load test due to the calibration of the jack or to the chosen failure criterion. 5. The error due to construction activities such as inadvertant batter, order of driving. The coefficients of variation presented in this article, and summarized in Table 8, correspond to the cummulation of errors 1 and 2.

Another part of the study dealt with the development of a design method for driven piles in sands which would include residual stresses (1). In the proposed method the ultimate capacity of the pile Q_u is calculated using the pile load test data base and correlations with the SPT blow count N:

$$Q_U = \left[A_p \times 19.75 \ (N)^{0.36} \right] + \left[A_f \times 0.224 \ (N)^{0.29} \right] \ \ldots \ \ldots \ (5)$$

TABLE 8.- Summary of Coefficients of Variation

Test	Coefficient of Variation (Horizontal)		Coefficient of Variation (Vertical)	
	Range	Average	Range	Average
SPT	0.144-0.770	0.421	0.164-1.148	0.707
PMT	0.282-0.682	0.416 (E) 0.399 (p_L)	0.516-0.783 0.261-0.575	0.619 (E) 0.503 (p_L)
CPT		Same order of magnitude as SPT and PMT		
SWVT		0.259		0.327
Density		0.057		

where A_p is the area of the pile point
              A_f is the area of the pile shaft

By using the same data base, the standard deviation of the ratio Q_u predicted over Q_u measured was calculated as 0.364 (1). The five errors mentioned earlier exist in the calculation of Q_u while errors 1 and 2 are involved in the value of N. It is of interest to find what portion of the error on Q_u is due to errors 1 and 2. Using the second order approximation of the Taylor series expansion for expected values, it comes:

$$Var\ (Q_u(N)) = \left[Q_u'(N) - \frac{1}{4} \left[(Q_u''(N) \right]^2 var(N) \right] Var(N) \ . \ . \ . \ (6)$$

This leads to a coefficient of variation for Q_u equal to 0.13 for a 1 ft square, 50 ft long pile. These calculations show that errors 1 and 2 account for 36% of the total error on Q_u.

Conclusion

The statistical analysis of the results of 92 borings showed that at a given site the precision on the soil parameters measured with the SPT, PMT and CPT is approximately the same and that only the cross-hole shear wave velocity shear modulus shows an increased precision. However, the repeatability of the tests from one site to another and from one operator to another is not included in the above analysis and it is argued that the rating of repeatability of these tests would be; 1. Cross-hole shear wave velocity and cone penetrometer, 2. Pressuremeter, 3. Standard Penetration Test. Other factors not included in the above analysis and important to consider before choosing one test over another are whether the soil parameter measured is representative of the phenomenon to be predicted, whether the test is cost effective and whether the test can be performed in all soil conditions.

The coefficient of variation of the soil parameter was shown to increase with the size of the area tested and with the depth of testing. Also the coefficient of variation of the soil parameter in the vertical direction was equal to 1.5 times that in the horizontal direction on the average. It was shown in one specific example that there is little advantage to carrying out more than 6 SPT borings at one site.

Five errors involved with the prediction of ultimate pile capacity were identified. The errors due to natural soil heterogeneity and testing procedures accounted for 36% of the error in the prediction of pile capacity for the chosen method.

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